March 13, 2018

Probability using Combinations

Case 1

There are 8 Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.
If you randomly pick three, without replacement, find the probability that you have three Whites.

P(3W) =

or

Case 1

There are 8 Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.
If you randomly pick three, without replacement, find the probability that you have three Whites.

$$P(3W) = \frac{3}{8} * \frac{2}{7} * \frac{1}{6} = \frac{6}{336} = 0.01786 = 0.018$$

or $$\frac{3C3}{8C3} = \frac{1}{56} = 0.01786 = 0.018$$

Case 2

There are 8 Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.
If you randomly pick three, without replacement, find the probability that you have two **W**hite and one **B**lack.

P(2W1B) =

or

Case 2

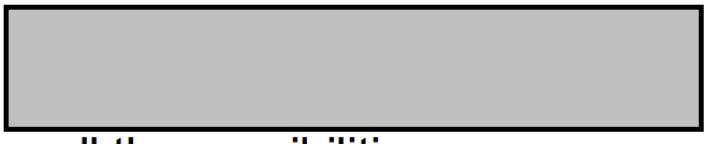
There are 8 Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.
If you randomly pick three, without replacement, find the probability that you have two **W**hite and one **B**lack.

$$P(2W1B) = \frac{3}{8} * \frac{2}{7} * \frac{5}{6} = \frac{30}{336} = 0.08929 = 0.089$$

or

$$\frac{3\,C\,2 * 5\,C\,1}{8\,C\,3} = \frac{15}{56} = 0.2679 = 0.27$$

What the...? These are not the same! Which is correct?
How can you find out?
Why are they different?
What is different between Case 1 and Case 2?

If you solved it using: 

then you didn't consider all the possibilities.

If you solved it using: 

then you either have a good understanding or you got lucky.

How can you be sure of solving this correctly?

You could draw the full sample space and count them.

If the black guinea pigs are A to E and the white guinea pigs are 1, 2, and 3, then this is the sample space without duplicates.

There are 56 in total.

How many arrangements have 2 white and 1 black?

ABC					
ABD	ACD				
ABE	ACE	ADE			
AB1	AC1	AD1	AE1		
AB2	AC2	AD2	AE2	A12	
AB3	AC3	AD3	AE3	A13	A23

	BCD				
	BCE	BDE			
	BC1	BD1	BE1		
	BC2	BD2	BE2	B12	
	BC3	BD3	BE3	B13	B23

		CDE			
		CD1	CE1		
		CD2	CE2	C12	
		CD3	CE3	C13	C23

			DE1		
			DE2	D12	
			DE3	D13	D23

				E12	
				E13	E23

					123

Ouch!

That's a lot of work, and how do you know if you got them all?

Is there an easier way? You bet.

Using the simplified sample space:

S = { 3B, 2B1W, 1B2W, 3W }

Solving by Permutations:

3B	BBB	= 5P3	= 60
2B1W	BBW or BWB or WBB	= 5P2 * 3P1 * 3	= 180
1B2W	BWW or WBW or WWB	= 5P1 * 3P2 * 3	= 90
3W	WWW	= 3P3	= 6
		Total	= 336

P(1B2W) = 90 / 8P3 = 90 / 336 = 0.27

Solving by Combinations:

3B	= *5C3*	= 10
2B1W	= 5C2 * 3C1	= 30
1B2W	= 5C1 * 3C2	= 15
3W	= 3C3	= 1
	Total	= 56

P(1B2W) = 15 / 8C3 = 15 / 56 = 0.27

P(2WB) =

Ready to try some more?

Case 3

There are 8 baby Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.

If you randomly pick three, without replacement, find the probability that you have one White and two Black.

P(1W2B) =

Case 3

There are 8 baby Guinea Pigs in a box at the pet store, 5 **B**lack and 3 **W**hite.

If you randomly pick three, without replacement, find the probability that you have one White and two Black.

$$P(1W2B) = \frac{5\ C\ 2 * 3\ C\ 1}{8\ C\ 3} = \frac{30}{56} = 0.5357 = 0.54$$

Case 4

There are 8 baby Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.

If you randomly pick three, without replacement, find the probability that you have selected
at least one **W**hite.

P(at least 1**W**) =

Case 4

There are 8 baby Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.

If you randomly pick three, without replacement, find the probability that you have selected
at least one **W**hite.

P(at least 1**W**) = 0.82

Remember: P(at least 1**W**) = 1 – P(No White)

No White is the same as 3 Black

$$P(3B) = \frac{5\,C\,3}{8\,C\,3} = \frac{10}{56} = 0.1786 = 0.18$$

P(at least 1W) = 1 – 0.18 = 0.82

Case 5

There are 8 baby Guinea Pigs in a box at the pet store,
5 **B**lack and 3 **W**hite.

If you randomly pick two, without replacement, find the probability that your second guinea pig is white given your first pick was black.

P(2^{nd} is W | 1^{st} was B) =

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